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INFLUENCE OF HEAT CONDUCTION ON THE UNLIMITED SHOCKLESS COMPRESSION OF A FLAT GASEOUS LAYER

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A numerical investigation is made of the influence of heat conduction on the compression of a flat gaseous layer in which, in the case of an ideal adiabatic gas, shockless compression occurs with unlimited cumulation of density and energy. Approximate formulas are obtained that characterize the asymptotic behavior of the cumulation of energy and density and of the energy expended on compression. It is shown that the phenomenon of density and energy cumulation is preserved when heat conduction is taken into account.

Introduction. The investigation of the gas-dynamic processes accompanying the unlimited growth (cumulation) of the density and temperature of a substance [1] is important for the study, for example, of thermonuclear fusion [2], the properties of matter under extreme conditions [3], etc. From the standpoint of minimizing the energy consumed in attaining the required values of the parameters, regimes in which shockless compression of matter is accomplished are the most interesting. For the plane and spherically symmetric cases, solutions of the gas-dynamic equations describing the adiabatic compression of an ideal gas with unlimited cumulation were given in [4, 5]. The recent results of [6–9] have stimulated interest in this problem. It turned out that it is not only one-dimensional flows that have the property of unlimited cumulation. From the analytical solutions constructed [7], describing the two- and three-dimensional, shockless compression of an ideal gas, it follows that the use of a compressing piston of a special shape or the organization of special three-dimensional boundary conditions enable one to obtain higher cumulation than in the spherically symmetric case. We note that the analytical solutions obtained can serve as a good test in the development and checking out of mathematical procedures and programs for calculating two- and three-dimensional gas-dynamic flows [10–13]. At the same time, the question of the influence of the actual properties of physical media on the process of shockless compression with cumulation remains little studied.

The present paper is devoted to an investigation of the influence of the thermal conductivity of the gas. It is well known that at temperatures exceeding 1 keV, which are typical of experiments on laser compression of microtargets [14], radiative and electronic mechanisms of heat transfer become important [15]. The simplest plane flow in which unlimited shockless compression occurs for an ideal adiabatic gas has been considered (the problem of the compression of a flat gaseous layer [4]). The effect of heat conduction was investigated numerically using the "Tigr" package of mathematical programs [16, 10, 11]. Along with its relative simplicity, the problem under consideration reflects the main aspects of the cumulation phenomenon and is convenient in that it has an exact analytical solution in the adiabatic case, which provides an additional control on the reliability of the calculations, and also facilitates the comparative analysis of the results with and without allowance for heat conduction. Numerical studies showed that if heat conduction is taken into account, the unlimited increase in the density of the material is retained although the average compression of the gas considerably decreases.

1. Formulation of the Problem. To investigate the influence of heat conduction on the regime of shockless gas compression, we consider the following problem.

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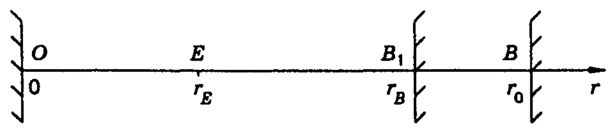

Fig. 1. Gas regions: OB is the gas region at the initial time and OB_1 is the gas region at an intermediate time, consisting of the region OE of undisturbed gas and the region EB_1 of the centered compression wave.

At the initial time $t = 0$, an ideal, nonheat-conducting gas with adiabatic index γ, speed of sound at rest c_0, and initial density ρ_0 lies between two parallel impenetrable planes that pass through the points O and B perpendicular to the r axis (Fig. 1). At $t = 0$, the thickness of the gas layer is $r_0 = |OB|$.

The point O is stationary. If the motion of the point B satisfies the law

$$r_B = \frac{\gamma+1}{\gamma-1} c_0 t_c^{(\gamma-1)/(\gamma+1)}(-\tau)^{2/(\gamma+1)} + \frac{2}{\gamma-1} c_0 \tau, \qquad \tau = t - t_c$$

or

$$u_B = \frac{dr_B}{dt} = \frac{2c_0}{\gamma-1}\left(1 - \left(\frac{-\tau}{t_c}\right)^{(1-\gamma)/(\gamma+1)}\right), \tag{1.1}$$

the gas is compressed without a shock wave and "collapses" at time $t_c = r_0/c_0$ (the collapse time). This problem (ignoring heat conduction) has an analytical solution [4]. At an intermediate time $0 < t < t_c$, the solution consists of a region OE of undisturbed gas ($r_E = -\tau c_0$) and a region EB_1 of a centered compression wave. The point B_1 corresponds to the position of the piston at time t. In the region B_1E we have

$$\begin{gathered} \rho = \rho_0\left(\frac{c}{c_0}\right)^{2/(\gamma-1)}, \qquad T = \frac{c^2}{C_V(\gamma-1)\gamma}, \qquad u = \frac{2}{\gamma-1}(c - c_0), \\ P = P_0\left(\frac{c}{c_0}\right)^{2\gamma/(\gamma-1)}, \qquad c = \left(c_0 + \frac{(1-\gamma)r}{2\tau}\right)\frac{2}{\gamma+1}, \qquad c_B = c_0\left(\frac{-\tau}{t_c}\right)^{(1-\gamma)/(\gamma+1)}, \end{gathered} \tag{1.2}$$

where ρ is the density, T is the temperature, u is the velocity, P is the pressure, c is the current value of the speed of sound in the gas, c_B is the speed of sound near the moving boundary of the gas; C_V is the specific heat at constant volume; quantities at the initial time are denoted by the subscript 0. This analytical solution for one-dimensional gas flow with cumulation was compared with a numerical solution in which heat conduction was taken into account.

One-dimensional gas flow with allowance for heat conduction is modeled by the equations

$$\begin{gathered} \frac{\partial u}{\partial t} + \frac{\partial P}{\partial q} = 0, \quad \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) - \frac{\partial u}{\partial q} = 0, \quad \frac{\partial r}{\partial t} = u, \quad \frac{\partial \varepsilon}{\partial t} + P\frac{\partial u}{\partial q} = \frac{\partial}{\partial q}\left(æ\rho\frac{\partial T}{\partial q}\right), \\ P = P(\rho, T), \qquad \varepsilon = \varepsilon(\rho, T), \qquad æ = æ(\rho, T), \end{gathered} \tag{1.3}$$

where $dq = \rho dr$ is the mass of a volume element, æ is the thermal-conductivity coefficient, and ε is the specific internal energy. The gas is assumed to be ideal in this case, but the contribution of equilibrium emission is taken into account, i.e., the following equations of state are used:

$$P = (\gamma-1)C_V\rho T + \frac{\sigma}{3}T^4, \qquad \varepsilon = C_V T + \frac{\sigma}{\rho}T^4,$$

where σ is the Stefan–Boltzmann constant, which equals zero in calculations without allowance for heat conduction.

The thermal-conductivity coefficient æ is taken in accordance with the Compton mechanism of photon scattering, which is the leading mechanism for light gases at temperatures above 1 keV [15],

$$æ = \frac{2}{\gamma-1}\sigma c_{\text{light}} a \frac{T^3}{\rho},$$

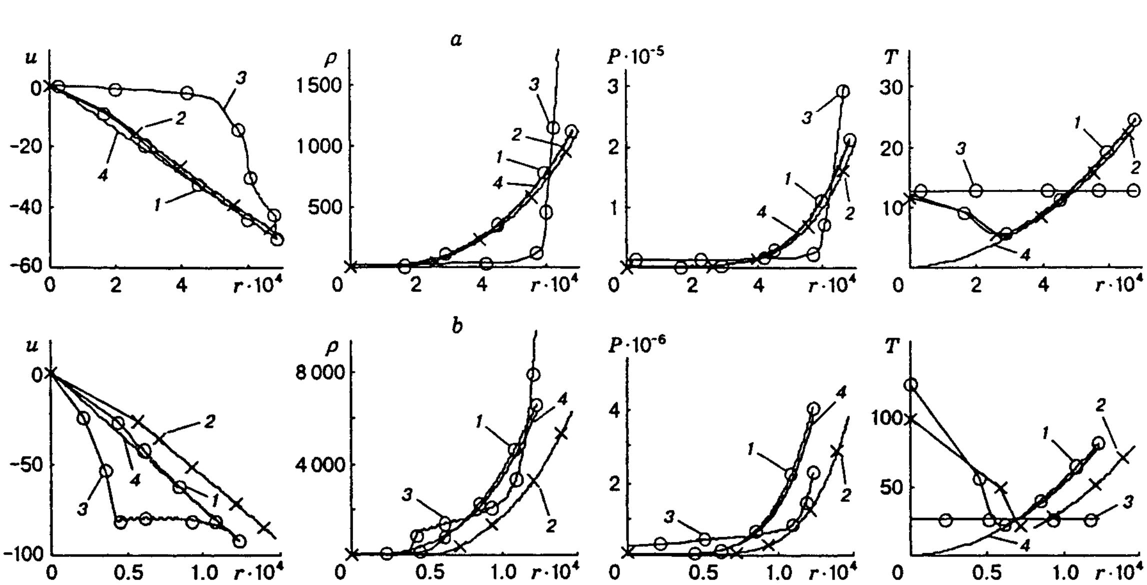

Fig. 2. Profiles of velocity, density, pressure, and temperature for times: $t = 0.99999$ (a) and $t = 0.999999$ (b); curves 1, 2, and 3 refer to the wv, wp, and hv calculations, respectively, and curve 4 refers to the analytical solution (ignoring heat conduction).

where c_{light} is the speed of light, a is a constant that depends on the choice of the system of units ($æ = 0$ in calculations ignoring heat conduction).

The following measurement units were adopted in the problem: r [cm], t [10^{-7} sec], q [g], and T [keV]. The calculations were made for $\sigma = 1.37$, $c_{\text{light}} = 3000$, $a = 5$, $\gamma = 5/3$, and $C_V = 11.58$ and initial conditions $r_0 = 1$, $\rho_0 = 0.2$, and $c_0 = 1$. Such values correspond to a DT mixture having the initial density of DT ice and a radius of 1 cm, which are of practical interest for inertial thermonuclear fusion [1].

2. Calculation Procedure. The calculations were made by the "Tigr" method [16], designed for solving two-dimensional problems of gas dynamics with allowance for heat conduction. The method uses an Eulerian–Lagrangian description of motion. One family of coordinate lines consists of a fixed set of straight lines along which nodes of the difference grid move, forming a Lagrangian family of coordinate lines. Such a coordinate system enables one to model with good accuracy the complex motions of multilayer systems with large deformations and to trace the contact discontinuities. The "Tigr" method also has a high economical efficiency because the difference equations are solved using implicit splitting schemes, both in coordinate directions and in physical processes.

The "Tigr" program package has been well recommended for three decades in the solution of an extensive class of practical problems. There are now several procedures that have been combined into the single "Tigr-VK" computer package [10, 11]. In the present work, we used a one-dimensional analog of this package, adjusted for the solution of Eqs. (1.3) in the Lagrangian coordinate system.

The calculations were made on a difference grid with 250 points along the spatial variable up to a time $t = 0.999999$, when the average compression in the analytical solution is $\approx$8000 while the maximum compression (near the piston) is $\approx$30,000 (the collapse time in our formulation is $t_c = 1$). The boundary condition at the point B (Fig. 1), which controls the gas compression, was given by formula (1.1) in the form of velocity (a "hard piston") or by formula (1.2), in which we took $c = c_B$, in the form of pressure (a "soft piston").

In the calculations with allowance for heat conduction, we had no right to expect unlimited shockless compression, since this requires a different law of motion of the compressing piston. Nevertheless, the influence of heat conduction on gas compression could be investigated.

We made calculations without allowance for heat conduction with velocity (wv) and pressure (wp) boundary conditions and calculations with allowance for heat conduction — (hv) and (hp), respectively.

TABLE 1

Results of Calculations for t = 0.99999

Parameters of the process	wv	wp	hv	hp	Analytical solution
r_B	$6.8089 \cdot 10^{-4}$	$7.0084 \cdot 10^{-4}$	$6.8099 \cdot 10^{-4}$	$3.7803 \cdot 10^{-2}$	$6.8131 \cdot 10^{-4}$
ρ^*	1129.6	1121.8	8493.4	6891.3	1124.7
ρ	293.72	285.37	293.69	5.2906	293.55
E	38.955	38.855	55.264	22.123	37.947
K	164.24	164.09	150.38	172.50	165.03
A	203.08	202.82	205.49	194.47	202.80

TABLE 2

Results of Calculations for t = 0.999999

Parameters of the process	wv	wp	hv	hp	Analytical solution
r_B	$1.2335 \cdot 10^{-4}$	$1.4696 \cdot 10^{-4}$	$1.2317 \cdot 10^{-4}$	$3.7288 \cdot 10^{-2}$ $3.7312 \cdot 10^{-2*}$	$1.2349 \cdot 10^{-4}$
ρ^*	6599.2	6307.0	10661.0	96733.0 96165.0*	6324.6
ρ	1624.8	1360.9	1623.8	5.3637 5.3602*	1619.3
E	130.60	124.37	131.52	40.876 40.633*	120.00
K	569.62	546.26	697.14	589.90 590.05*	555.36
A	700.22	670.63	828.57	630.66 630.53*	675.18

Note. Asterisk refers to the calculation with twice the number of points in the difference grid.

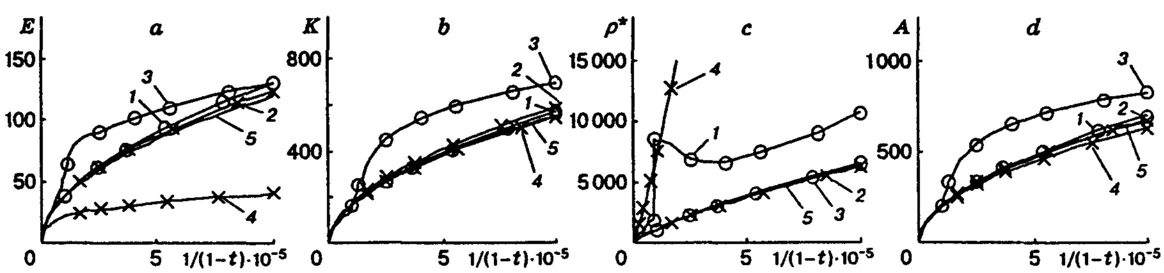

Fig. 3. Time dependence of internal (a) and kinetic (b) energy of the gas, maximum gas density (near the piston) (c), and work of the piston on compression (d): curves 1, 2, 3, and 4 refer to wv wp, hv, and hp calculation, and curve 5 refers to the analytical solution (without allowance for heat conduction).

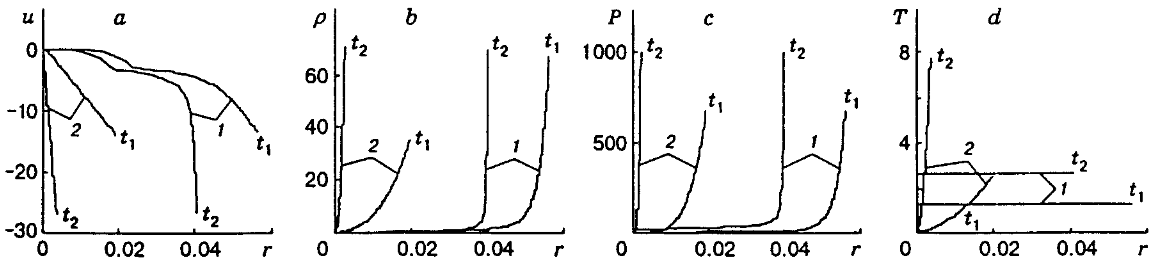

Fig. 4. Profiles of velocity (a), density (b), pressure (c), and temperature (d) at the times $t_1 = 0.999$ and $t_2 = 0.9999$: curves 1 and 2 refer to the hp calculation and analytical solution (without allowance for heat conduction), respectively.

The results of the calculations in comparison with the analytical solution of (1.1),(1.2) are given in graphs and tables.

3. Discussion. The wv and wp calculations were made to estimate the error of the numerical solution. On the Lagrangian grid, the domain OE of undisturbed gas (Fig. 1) disappears as soon as its size becomes comparable to the grid step size h, i.e., at $t > 1 - h = 0.996$. This results in an increase in the gas-dynamic quantities and temperature near the stationary wall and violation of isentropy, in contrast to the analytical solution. In Figure 2 shows profiles of the main gas-dynamic quantities for two times, and their numerical values in comparison to the analytical solutions are given in Tables 1 and 2, in which r_B is the position of the piston, ρ^* is the maximum density, ρ is the average density, E and K are the internal and kinetic energies of the gas, and A is the work of the piston in compression. At the later calculation time (Fig. 2b), the region of strong departure of the temperature from the exact solution (near the left boundary) is two or three points of the Lagrangian grid. The concentration of points increases as the right boundary of the gas region is approached. To reproduce the analytical solution in the entire gaseous region, one must use a dynamically adaptive difference grid that is made finer at both boundaries as necessary. Because of the loss of the region of undisturbed gas in the wp calculation with a "soft piston," we observe lagging of the moving boundary due to the opposing action of internal pressure forces. At $t = 0.99999$ (Fig. 2a and Table 1) the lag is 3%, and at $t = 0.999999$ (Fig. 2b and Table 2) it is 25%. Against the background of high gradients of the gas-dynamic quantities near the piston, these departures do not significantly affect the process of unlimited, shockless gas compression. The relative error of the maximum density near the piston at $t = 0.99999$ is 0.44% (wv calculation) and 0.26% (wp calculation), and at $t = 0.999999$ it is 4.34% (wv calculation) and 0.28% (wp calculation). Despite the deep stage of compression, the calculation accuracy in the cumulation region remains satisfactory, which inspires confidence that calculations with allowance for heat conduction will have the same accuracy.

To estimate the error associated with the step size of the spatial grid, we repeated the hp calculation on a grid with twice as many points (see Table 2). The error is tenths of a percent, which indicates that a sufficiently detailed difference grid was chosen.

In calculations with allowance for heat conduction, the gas temperature is equalized over space. The region of "undisturbed" gas is heated, which leads to a pressure increase at the stationary wall. At $t \approx 0.9$, a shock wave is formed, and the region of "undisturbed" gas is about twice as large as in the problem without allowance for heat conduction.

In the hv calculation with a "hard piston," the shock wave moves toward the stationary wall, is reflected from it (at $t \approx 0.999$), and at $t \approx 0.9999905$, it collides again with the piston. In the process, a local density maximum ρ^* is formed near the piston. Figure 3 gives integral characteristics of the process and the maximum density as functions of time. The energy consumed in compression at the final time in the hv calculation is 23% higher than in the analytical solution in which heat conduction is not taken into account. The maximum density (near the piston) is 69% higher (see Table 2).

In the hv calculation with a "hard piston," nothing develops to prevent the advance of the piston. The

TABLE 3

Parameters of the process	hp	Analytical solution
ρ^*	$0.0126\cdot(-\tau)^{-1.1473}$	$0.2000\cdot(-\tau)^{-0.7500}$
ρ	$4.9398\cdot(-\tau)^{-0.0060}$	$0.0574\cdot(-\tau)^{-0.7417}$
E	$1.0273\cdot(-\tau)^{-0.2666}$	$0.1200\cdot(-\tau)^{-0.5000}$
K	$0.3688\cdot(-\tau)^{-0.5340}$	$0.3824\cdot(-\tau)^{-0.5270}$
A	$0.5421\cdot(-\tau)^{-0.5109}$	$0.4957\cdot(-\tau)^{-0.5224}$

gas can thus be compressed to any density. A hp calculation with allowance for heat conduction and with a pressure boundary condition is therefore of the greatest practical interest.

In the hp calculation with a "soft piston," a shock wave is formed in the initial stage, as in the hv calculation. The opposing action of internal pressure forces then retards the motion of the right-hand gas boundary, while the shock wave weakens. Figure 4 gives profiles of the gas-dynamic quantities in the hp calculation (with heat conduction) in comparison with the analytical solution (without heat conduction) at times t_1 and t_2. At $t = 0.999999$, the piston's coordinate is 300 times larger than in the analytical solution (see Table 2). But the density near the piston is 15 times higher because of the temperature decrease in the cumulation region. Here the gas is almost entirely concentrated near the moving boundary. Thus, 87.6% of the mass of the gas is found in a volume of 0.02% of the gaseous region (near the piston) at the final time.

Estimates of the degree of cumulation of gas density, pressure, and energy and the energy consumed in compression are also of practical interest. Estimates of the degree of energy and pressure cumulation for an ideal adiabatic gas were given in [5]. In the plane case, the work of pressure forces on gas compression and the internal and kinetic energies are calculated from the equations

$$A = \int_0^t Pu\,dt = \rho_0 \frac{2(\gamma+1)}{\gamma(\gamma-1)^2}[(0.5\tau^2 - 1)(-\tau)^{(1-\gamma)/(\gamma+1)} + 0.5],$$

$$E = \int_0^r \frac{P}{(\gamma-1)}\,dr = \frac{\rho_0(\gamma+1)}{\gamma(3\gamma-1)(\gamma-1)}[(-\tau)^{-2(\gamma-1)/(\gamma+1)} + \tau] - \tau\frac{\rho_0}{(\gamma-1)\gamma}, \quad K = A - E + E_0,$$

where $E_0 = \rho_0/(\gamma-1)\gamma$ is the initial internal energy. Approximate formulas for these quantities in the hp calculation (with allowance for heat conduction) and in the analytical solution (ignoring heat conduction), as well as for the maximum density ρ^* and the average density ρ, which determine the asymptotic behavior of the solution and the degree of cumulation near $t = 0.999999$, obtained by choosing the coefficients from the results of the hp calculation, are given in Table 3 in the form of exponential functions. It is seen that the degree of density cumulation in the calculation with allowance for heat conduction, with less energy consumed in compression, is 1.5 times higher than in the analytical solution (ignoring heat conduction).

Conclusion. Allowance for heat conduction in adiabatic flow with cumulation leads to temperature equalization, so that the gas becomes nearly isothermal in the spatial variable. Despite the fact that the average gas compression is considerably degraded, the maximum compression near the piston is far higher than in the analytical solution (ignoring heat conduction), and the main mass of the gas has a high density. The phenomenon of density cumulation is thus preserved.

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